SAT-IT: the Interactive SAT-Tracer

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Motivation and main contributions

Provide interactive tool for better **learning and understanding basic SAT-solving algorithms**

- Backtracking
- Davis-Putnam-Logemann-Loveland (DPLL)
- Conflict-Driven Clause Learning (CDCL)

Provide interactive tool for better **analysing encodings properties**

Context of the course where it is used (1/2)

Subject: **Declarative Programming, Applications**

- 4th year undergraduate students in computer science
- Previous knowledge: Complexity (NP-completeness), Prolog, data-structures and advanced programming, etc.
- 5 ECTS = 50 hours
- TWO parts
  - First part of functional programming (20h)
  - **Second part of combinatorial problem solving (30h)**

Context of the course where it is used (2/2)

Combinatorial problem solving

- Constraint Programming (MiniZinc) [20 hours]
- **SAT solving and modelling (ad-hoc SCALA API) [10 hours]**
  - Backtracking, DPLL and CDCL
  - Cardinality constraints, PB constraints, ...

  Assignment:
  - use of given algorithms for solving Formulas
  - implement some cardinality constraints encodings
  - model and solve: Binary Sudoku, Crowded chessboard, Minesweeper, etc...

Boolean Formulas

A **Formula** is typically represented in **Conjunctive Normal Form (CNF)**

- **Variables**: e.g. $x_1, x_2, ...$
- **Literals** (variables and negated variables): e.g. x_1 and $\neg x_1$
- **Clauses** (disjunctions of literals): $(x_1 \vee \neg x_2 \vee x_3)$
- **CNF** formulas (conjunctions of clauses):
  $(x_1 \vee x_2) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2)$

An **interpretation** assigns truth values to variables, e.g.:
$x_1 \rightarrow \mathit{false}, x_2 \rightarrow \mathit{false}, x_3 \rightarrow \mathit{true}$, represented as: $\neg x_1 \neg x_2 x_3$

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An **interpretation** assigns truth values to variables, e.g.:
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- A **clause is satisfied by an interpretation** if some of its literals appears in the interpretation
- A **CNF is satisfied by an interpretation (model)** if all of its clauses are satisfied by the interpretation

$$\neg x_1 x_2 x_3 \models (x_1 \vee x_2) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2)$$

$$x_1 x_2 x_3 \not\models (x_1 \vee x_2) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2)$$

SAT as *"Machine Code"* for Combinatorial Problem Solving

SAT is the problem of determining the satisfiability of a Boolean Formula, i.e. the existence of an interpretation that satisfies the formula.

- It is **"the" NP-Complete problem**
- It is a sort of *"Machine Code"* for Combinatorial Problem Solving

SAT is an elemental CSP

- Set of variables: $\{x_1, x_2, x_3\}$
- With Domains: $dom(x_1) = dom(x_2) = dom(x_3) = \{false, true\}$
- Set of constraints = set of clauses:
  $\{(x_1 \vee x_2), (x_1 \vee \neg x_2 \vee x_3), (\neg x_1 \vee x_2), (\neg x_1 \vee \neg x_2)\}$

The Trail System

In our course we follow a symbolic representation of the evolution of the searching process borrowed and adapted from:

Robert Nieuwenhuis, Albert Oliveras and Cesare Tinelli.
Solving SAT and SAT Modulo Theories: From an abstract Davis-Putnam Logemann Loveland procedure to DPLL(T).
In *Journal of the ACM*, 53(6): 937-977, 2006.

- The state is represented as a pair:

  (**annotated**) partial assignment$\|$ (**possibly updated**) original formula

  e.g.: $x_1^1\ x_2^k \parallel \{(x_1 \vee x_2), (x_1 \vee \neg x_2 \vee x_3), (\neg x_1 \vee x_2), (\neg x_1 \vee \neg x_2)\}$
- The **rules transform the state until a model is found or unsatisfiability has been proved** (there are NO decisions to change)
- **Aplycability of rules** is defined in terms of properties of the partial assignment and the formula

Backtracking Rules

Decision level: counter on amount of decisions present in the current trail

---

DECIDE:

$$M \parallel F \quad \Longrightarrow M\ l^d \parallel F \quad \textbf{if} \begin{cases} l \text{ or } \neg l \text{ occurs in a clause of } F \\ l \text{ is undefined in } M \end{cases}$$

Backtracking Rules

Decision level: counter on amount of decisions present in the current trail

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BACKTRACK:

$$M\ l^d\ N \parallel F \cup \{C_i\} \quad \Longrightarrow M\ \neg l^k \parallel F \cup \{C_i\} \quad \textbf{if} \begin{cases} M\ l^d\ N \models \neg C_i \\ N \text{ contains no decision literals} \end{cases}$$

Backtracking Rules

Decision level: counter on amount of decisions present in the current trail

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FAIL:

$$M \parallel F \cup \{C_i\} \quad \Longrightarrow \mathit{FailState} \quad \textbf{if} \begin{cases} M \models \neg C_i \\ M \text{ contains no decision literals} \end{cases}$$

Backtracking Algorithm

Algorithm 1: BACKTRACKING algorithm

Input: $F = \{C_1, \ldots, C_{|F|}\}$, a set of clauses.
Output: If F is SAT: (SAT, M), where M is a model of F.
Otherwise: $UNSAT$.

```
1 M ← {}
2 while True do
3     if CanApply(Backtrack,M,F) then (M, F) ←Backtrack(M,F) ;
4     else if CanApply(Fail,M,F) then return UNSAT ;
5     else if M is a complete assignment then return (SAT,M) ;
6     else (M, F) ←Decide(M,F) ;
```

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$$C1 : (x_1 \vee x_2)$$
$$C2 : (x_1 \vee \neg x_2)$$
$$C3 : (\neg x_1 \vee x_2)$$
$$C4 : (\neg x_1 \vee \neg x_2)$$

Backtracking Example

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x_1^d

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$x_1^d x_2^d$

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$x_1^d x_2^d$

Fail C4, Backtrack

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$C1 : (x_1 \vee x_2)$

$C2 : (x_1 \vee \neg x_2)$

$C3 : (\neg x_1 \vee x_2)$

$C4 : (\neg x_1 \vee \neg x_2)$

$x_1^d x_2^d$

$x_1^d \neg x_2^k$

Fail C4, Backtrack

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$C1 : (x_1 \vee x_2)$	$x_1^d x_2^d$	Fail C4, Backtrack
$C2 : (x_1 \vee \neg x_2)$	$x_1^d \neg x_2^k$	Fail C3, Backtrack
$C3 : (\neg x_1 \vee x_2)$		
$C4 : (\neg x_1 \vee \neg x_2)$		

Backtracking Example

We set an order in the decision of variables:

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$C1 : (x_1 \vee x_2)$	$x_1^d x_2^d$	Fail C4, Backtrack
$C2 : (x_1 \vee \neg x_2)$	$x_1^d \neg x_2^k$	Fail C3, Backtrack
$C3 : (\neg x_1 \vee x_2)$	$\neg x_1^k$	
$C4 : (\neg x_1 \vee \neg x_2)$		

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$C1 : (x_1 \vee x_2)$	$x_1^d x_2^d$	Fail C4, Backtrack
$C2 : (x_1 \vee \neg x_2)$	$x_1^d \neg x_2^k$	Fail C3, Backtrack
$C3 : (\neg x_1 \vee x_2)$	$\neg x_1^k x_2^d$	
$C4 : (\neg x_1 \vee \neg x_2)$		

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$C1 : (x_1 \vee x_2)$	$x_1^d x_2^d$	Fail C4, Backtrack
$C2 : (x_1 \vee \neg x_2)$	$x_1^d \neg x_2^k$	Fail C3, Backtrack
$C3 : (\neg x_1 \vee x_2)$	$\neg x_1^k x_2^d$	Fail C2, Backtrack
$C4 : (\neg x_1 \vee \neg x_2)$		

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$C1 : (x_1 \vee x_2)$	$x_1^d x_2^d$	Fail C4, Backtrack
$C2 : (x_1 \vee \neg x_2)$	$x_1^d \neg x_2^k$	Fail C3, Backtrack
$C3 : (\neg x_1 \vee x_2)$	$\neg x_1^k x_2^d$	Fail C2, Backtrack
$C4 : (\neg x_1 \vee \neg x_2)$	$\neg x_1^k \neg x_2^k$	

Backtracking Example

We set an order in the decision of variables:

$$x_1 \prec x_2$$

$C1 : (x_1 \vee x_2)$	$x_1^d x_2^d$	Fail C4, Backtrack
$C2 : (x_1 \vee \neg x_2)$	$x_1^d \neg x_2^k$	Fail C3, Backtrack
$C3 : (\neg x_1 \vee x_2)$	$\neg x_1^k x_2^d$	Fail C2, Backtrack
$C4 : (\neg x_1 \vee \neg x_2)$	$\neg x_1^k \neg x_2^k$	Fail C1, UNSAT

Davis-Putnam-Logemann-Loveland (DPLL) Rules

DECIDE:

$$M \parallel F \quad \Longrightarrow M\ l^d \parallel F \quad \textbf{if} \begin{cases} l \text{ or } \neg l \text{ occurs in a clause of } F \\ l \text{ is undefined in } M \end{cases}$$

BACKTRACK:

$$M\ l^d\ N \parallel F \cup \{C_i\} \quad \Longrightarrow M\ \neg l^k \parallel F \cup \{C_i\} \quad \textbf{if} \begin{cases} M\ l^d\ N \models \neg C_i \\ N \text{ contains no decision literals} \end{cases}$$

FAIL:

$$M \parallel F \cup \{C_i\} \quad \Longrightarrow \mathit{FailState} \quad \textbf{if} \begin{cases} M \models \neg C_i \\ M \text{ contains no decision literals} \end{cases}$$

UNITPROPAGATE :

$$M \parallel F \cup \{C_i\} \quad \Longrightarrow M\ l^i \parallel F \cup \{C_i\} \quad \textbf{if} \begin{cases} C_i \text{ has the form } C' \vee l \\ M \models \neg C' \\ l \text{ is undefined in } M \end{cases}$$

DPLL Algorithm

Algorithm 2: DPLL algorithm

Input: $F = \{C_1, \ldots, C_{|F|}\}$, a set of clauses.
Output: If F is SAT: (SAT, M), where M is a model of F.
Otherwise: $UNSAT$.

```
1 M ← {}
2 while True do
3     if CanApply(Backtrack,M,F) then (M, F) ←Backtrack(M,F) ;
4     else if CanApply(Fail,M,F) then return UNSAT ;
5     else if M is a complete assignment then return (SAT,M) ;
6     else if CanApply(UnitPropagate,M,F) then
        (M, F) ←UnitPropagate(M,F) ;
7     else (M, F) ←Decide(M,F) ;
```

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$$
$$C2 : (\neg x_3 \vee x_4 \vee x_5)$$
$$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$$
$$C4 : (x_1 \vee x_2)$$
$$C5 : (\neg x_1 \vee \neg x_5)$$
$$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$$

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

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x_1^d

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$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5$

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$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5 \; x_2^d$

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Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

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$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5 \; x_2^d \; x_3^d$

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

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$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$$x_1^d \; \neg x_5^5 \; x_2^d \; x_3^d \; x_4^2$$

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5 \; x_2^d \; x_3^d \; x_4^2$

Fail C6, Backtrack

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5 \; x_2^d \; x_3^d \; x_4^2$ Fail C6, Backtrack

$x_1^d \; \neg x_5^5 \; x_2^d \; \neg x_3^k$

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

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$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5 \; x_2^d \; x_3^d \; x_4^2$ Fail C6, Backtrack

$x_1^d \; \neg x_5^5 \; x_2^d \; \neg x_3^k \; x_4^1$

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

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$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

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$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^5 \; x_2^d \; x_3^d \; x_4^2$ Fail C6, Backtrack

$x_1^d \; \neg x_5^5 \; x_2^d \; \neg x_3^k \; x_4^1$ Fail C3, Backtrack

$x_1^d \; \neg x_5^5 \; \neg x_2^k \; x_3^d$

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

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$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	

DPLL Example

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$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

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$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

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$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

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$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (\neg x_1 \vee \neg x_5)$
$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (\neg x_1 \vee \neg x_5)$
$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k x_2^4$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k x_2^4 x_3^d$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (\neg x_1 \vee \neg x_5)$
$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k x_2^4 x_3^d x_4^d$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (\neg x_1 \vee \neg x_5)$

$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k x_2^4 x_3^d x_4^d x_5^6$	

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (\neg x_1 \vee \neg x_5)$
$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k x_2^4 x_3^d x_4^d x_5^6$	SAT

DPLL Example

Decisions' order:

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (\neg x_1 \vee \neg x_5)$
$C6 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^5 x_2^d x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 x_2^d \neg x_3^k x_4^1$	Fail C3, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k x_3^d x_4^2$	Fail C6, Backtrack
$x_1^d \neg x_5^5 \neg x_2^k \neg x_3^k x_4^1$	Fail C3, Backtrack
$\neg x_1^k x_2^4 x_3^d x_4^d x_5^6$	SAT

The obtained model is $\neg x_1\, x_2\, x_3\, x_4\, x_5$

Conflict-Driven Clause Learning (CDCL) Rules

Decide:

$$M \parallel F \implies M\ l^d \parallel F \quad \textbf{if} \begin{cases} l \text{ or } \neg l \text{ occurs in a clause of } F \\ l \text{ is undefined in } M \end{cases}$$

Backjump:

$$M\ l^d\ N \parallel F \cup \{C_i\} \implies M\ l'^j \parallel F \cup \{C_i\} \quad \textbf{if} \begin{cases} M\ l^d\ N \models \neg C_i, \text{ and} \\ \text{there is some clause } C_j \text{ such that:} \\ \quad C_j \text{ has the form } C' \vee l', \\ \quad F \cup \{C_i\} \models C_j \text{ and } M \models \neg C', \\ \quad l' \text{ is undefined in } M, \text{ and} \\ \quad l' \text{ or } \neg l' \text{ occurs in } F \text{ or in } M\ l^d\ N \end{cases}$$

Fail:

$$M \parallel F \cup \{C_i\} \implies \mathit{FailState} \quad \textbf{if} \begin{cases} M \models \neg C_i \\ M \text{ contains no decision literals} \end{cases}$$

UnitPropagate :

$$M \parallel F \cup \{C_i\} \implies M\ l^i \parallel F \cup \{C_i\} \quad \textbf{if} \begin{cases} C_i \text{ has the form } C' \vee l \\ M \models \neg C' \\ l \text{ is undefined in } M \end{cases}$$

Learn:

$$M \parallel F \implies M \parallel F \cup \{C_j\} \quad \textbf{if} \begin{cases} \text{each literal of } C_j \text{ occurs in } F \text{ or in } M \\ F \models C_j \end{cases}$$

CDCL Algorithm

Algorithm 3: CDCL algorithm

Input: $F = \{C_1, \ldots, C_{|F|}\}$, a set of clauses.
Output: If F is SAT: (SAT, M), where M is a model of F.
Otherwise: $UNSAT$.

```
1  M ← {}
2  while True do
3      if CanApply(Backjump,M,F) then
4          C_j ←ConflictAnalysis(M,F)
5          (M, F) ←Backjump(M,F,C_j)
6          (M, F) ←Learn(M,F,C_j)
7      else if CanApply(Fail,M,F) then return UNSAT ;
8      else if M is a complete assignment then return (SAT,M) ;
9      else if CanApply(UnitPropagate,M,F) then
           (M, F) ←UnitPropagate(M,F) ;
10     else (M, F) ←Decide(M,F) ;
```

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$ $\quad x_1^d$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$ $\quad x_1^d \; \neg x_5^6$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^6 \; x_2^d$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^6 \; x_2^d \; x_3^d$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$ Fail C7

Conflict analysis:
(in blue, lits. of current decision level)

$$(\neg x_3 \vee \neg x_4 \vee x_5)$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (x_1 \vee \neg x_2)$
$C6 : (\neg x_1 \vee \neg x_5)$
$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$x_1^d \; \neg x_5^6 \; x_2^d \; x_3^d \; x_4^2$ Fail C7

Conflict analysis:
(in blue, lits. of current decision level)

$$\frac{(\neg x_3 \vee \neg x_4 \vee x_5) \qquad (\neg x_3 \vee x_4 \vee x_5)}{(\neg x_3 \vee x_5)}$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$C8 : (\neg x_3 \vee x_5)$

$x_1^d \; \neg x_5^6 \; x_2^d \; x_3^d \; x_4^2$ Fail C7 , Learn C8, Backjump

$x_1^d \; \neg x_5^6 \; \neg x_3^8$

Conflict analysis:
(in blue, lits. of current decision level)

$$\frac{(\neg x_3 \vee \neg x_4 \vee x_5) \qquad (\neg x_3 \vee x_4 \vee x_5)}{(\neg x_3 \vee x_5)}$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$C8 : (\neg x_3 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$

$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$

Fail C7 , Learn C8, Backjump

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$C8 : (\neg x_3 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$ Fail C7 , Learn C8, Backjump

$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$ Fail C3

Conflict analysis:
(in blue, lits. of current decision level)

$(x_3 \vee \neg x_4 \vee \neg x_1)$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (x_1 \vee \neg x_2)$
$C6 : (\neg x_1 \vee \neg x_5)$
$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$
$C8 : (\neg x_3 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$ Fail C7 , Learn C8, Backjump

$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$ Fail C3

Conflict analysis:
(in blue, lits. of current decision level)

$$\frac{(x_3 \vee \neg x_4 \vee \neg x_1) \qquad (x_3 \vee x_4 \vee \neg x_1 \vee x_5)}{(x_3 \vee \neg x_1 \vee x_5)}$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$C8 : (\neg x_3 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$ Fail C7 , Learn C8, Backjump

$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$ Fail C3

Conflict analysis:
(in blue, lits. of current decision level)

$$\dfrac{\dfrac{(x_3 \vee \neg x_4 \vee \neg x_1) \qquad (x_3 \vee x_4 \vee \neg x_1 \vee x_5)}{(x_3 \vee \neg x_1 \vee x_5)} \qquad (\neg x_3 \vee x_5)}{(\neg x_1 \vee x_5)}$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$C8 : (\neg x_3 \vee x_5)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$ Fail C7 , Learn C8, Backjump

$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$ Fail C3

Conflict analysis:
(in blue, lits. of current decision level)

$$\dfrac{\dfrac{\dfrac{(x_3 \vee \neg x_4 \vee \neg x_1) \qquad (x_3 \vee x_4 \vee \neg x_1 \vee x_5)}{(x_3 \vee \neg x_1 \vee x_5)} \qquad (\neg x_3 \vee x_5)}{(\neg x_1 \vee x_5)} \qquad (\neg x_1 \vee \neg x_5)}{(\neg x_1)}$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (x_1 \vee \neg x_2)$
$C6 : (\neg x_1 \vee \neg x_5)$
$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$
$C8 : (\neg x_3 \vee x_5)$
$C9 : (\neg x_1)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$ Fail C7 , Learn C8, Backjump
$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$ Fail C3 , Learn C9, Backjump
$\neg x_1^9$

Conflict analysis:
(in blue, lits. of current decision level)

$$\dfrac{\dfrac{\dfrac{(x_3 \vee \neg x_4 \vee \neg x_1) \qquad (x_3 \vee x_4 \vee \neg x_1 \vee x_5)}{(x_3 \vee \neg x_1 \vee x_5)} \qquad (\neg x_3 \vee x_5)}{(\neg x_1 \vee x_5)} \qquad (\neg x_1 \vee \neg x_5)}{(\neg x_1)}$$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$

$C2 : (\neg x_3 \vee x_4 \vee x_5)$

$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$

$C4 : (x_1 \vee x_2)$

$C5 : (x_1 \vee \neg x_2)$

$C6 : (\neg x_1 \vee \neg x_5)$

$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$

$C8 : (\neg x_3 \vee x_5)$

$C9 : (\neg x_1)$

$x_1^d \; \neg x_5^6 \; x_2^d \; x_3^d \; x_4^2$ Fail C7 , Learn C8, Backjump

$x_1^d \; \neg x_5^6 \; \neg x_3^8 \; x_4^1$ Fail C3 , Learn C9, Backjump

$\neg x_1^9 \; x_2^4$

CDCL Example

Unsatisfiable formula solved with CDCL and the following order for decisions

$$x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$$

$C1 : (x_3 \vee x_4 \vee \neg x_1 \vee x_5)$
$C2 : (\neg x_3 \vee x_4 \vee x_5)$
$C3 : (x_3 \vee \neg x_4 \vee \neg x_1)$
$C4 : (x_1 \vee x_2)$
$C5 : (x_1 \vee \neg x_2)$
$C6 : (\neg x_1 \vee \neg x_5)$
$C7 : (\neg x_3 \vee \neg x_4 \vee x_5)$
$C8 : (\neg x_3 \vee x_5)$
$C9 : (\neg x_1)$

$x_1^d \neg x_5^6 x_2^d x_3^d x_4^2$	Fail C7 , Learn C8, Backjump
$x_1^d \neg x_5^6 \neg x_3^8 x_4^1$	Fail C3 , Learn C9, Backjump
$\neg x_1^9 x_2^4$	Fail C5, UNSAT

Example of N-Queens and System demonstration

Place N queens in a N×N chessbord such that they don't threat each other (rows, columns and diagonals)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Variable i encodes the presence of queen in the corresponding cell

```
c exactly one queen per row
```
$1 \vee 2 \vee 3 \vee 4$
$\neg 1 \vee \neg 2$
$\neg 1 \vee \neg 3$
$\neg 1 \vee \neg 4$
$\neg 2 \vee \neg 3$
$\neg 2 \vee \neg 4$
$\neg 3 \vee \neg 4$
...
```
c exactly one queen per column
```
$1 \vee 5 \vee 9 \vee 13$
$\neg 1 \vee \neg 5$
$\neg 1 \vee \neg 9$
$\neg 1 \vee \neg 13$
$\neg 5 \vee \neg 9$
$\neg 5 \vee \neg 13$
$\neg 9 \vee \neg 13$
...
```
c exactly one queen per diagonal
```
$2 \vee 5$
$\neg 2 \vee \neg 5$
...

Future Work

- Move application to web
- Integrate it with our own declarative SAT modelling language (GOS/BUP)
- Extending rules (e.g. pure literal, restart, ...)
- Consider MAX-SAT algorithms
- ...

Thanks for your attention

Tool available at: `https://imae.udg.edu/Recerca/LAI/`